Chapter 2:  
Algebraic summary:  
A “macro-monetary” interpretation of Marx’s theory

This chapter summarizes the “macro-monetary-sequential” interpretation of Marx’s theory of the production and distribution of surplus-value presented in this book in algebraic form. It is hoped that this algebraic summary will clarify the main points and will serve as a reference point for later chapters, which will provide further arguments and extensive textual evidence for this interpretation.

1. Theory of the production of surplus-value

I argued in Chapter 1 that the general analytical framework of Marx’s theory of the production of surplus-value is the circuit of money capital:

\[ M - C \ldots P \ldots C' - M' \]

where \( M' = M + \Delta M \).

In Volume 1, this circuit of money capital applies to the total capital in the economy as a whole. The main question of Volume 1 is the determination of the magnitude of the total surplus-value produced in the economy as a whole. This aggregate circuit does not imply that all the individual capitals go through the different phases in tandem (obviously they do not), but only that they all go through these phases in this order – money capital is advanced and then more money capital is recovered, together with \( \Delta M \). All the individual capitals that go through these similar circuits over the course of a year can be added up to obtain the aggregate totals, which is the subject of Volume 1. Most of the variables discussed in this section refer to aggregate variables for the economy as a whole (all except equations 10-14).

The initial money capital \( M \) at the beginning of the circuit of money capital consists of two components – constant capital invested in means of production and variable capital invested in labor-power. Part of the advanced constant capital is fixed capital, which is
invested in long-lasting means of production and which is transferred to the value of the output “bit by bit” and thus recovered “bit by bit” by the annual depreciation charge over the expected lifetime of the means of production.\(^1\) If we consider a period of time of one year, then constant capital consists of the annual depreciation cost of fixed capital plus the circulating constant capital of that year (the cost of raw materials, auxiliary materials, etc.), and variable capital is the annual wage cost. The sum of these costs is the *cost price* of commodities.

The final money capital recovered \(M'\) in a year is equal to the value of the commodities produced and sold \((P)\). Therefore, the surplus-value produced in one year is equal to the difference between the value of the commodities produced during this year and the cost of producing these commodities \((K)\):

\[
(1) \quad S = P - K
\]

**Cost price**

We have just seen that the cost price of commodities is the sum of two components: constant capital \((C)\)\(^2\) and variable capital \((V)\):

\[
(2) \quad K = C + V
\]

I argued in Chapter 1 that constant capital and variable capital are *taken as given* in Marx’s theory of surplus-value, as the actual (equilibrium) quantities of money capital advanced to purchase means of production and labor-power (equal to the price of production and the

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\(^1\) The annual depreciation cost of constant capital of each type of means of production \((D)\) is equal to the total fixed constant capital invested in each type of means of production \((FC)\) divided by its expected lifetime \((Y, \text{ in years})\) (i.e. \(D = FC / Y\)). The fact that depreciation is computed in this way is itself evidence that constant capital (in this case fixed constant capital) is taken as given in Marx’s theory of value and surplus-value.

\(^2\) Please note that this \(C\) which stands for constant capital is not the same as the \(C\) which stands for commodities in the circuit of money capital. The context should make clear which \(C\) I am talking about in any passage.
means of production and means of subsistence, respectively). These actual quantities of money capital are advanced, and therefore they exist at the beginning of the circuit of capital, and can be taken as given as such, as the starting point of a process to be analyzed. The precise question of Marx’s theory of surplus-value is: how does this pre-existing and given quantity of money capital become a larger quantity of money capital (through the production and sale of commodities)? In order to indicate that constant capital and variable capital are taken as given, I will use a bar over these variables in the equations below, and equation (1) can be rewritten as follows:

\[
(1)' \quad S = P - (\bar{C} + \bar{V})
\]

This is the first sense in which constant capital and variable capital are taken as given – as components of the cost price that are subtracted from the value of the commodities produced in order to determine the quantity of surplus-value. As components of the cost price, there is no difference between constant capital and variable capital; both are costs which must be recovered before surplus-value can be appropriated. We will see below that these same quantities of constant capital and variable capital are also taken as given in the determination of the value and surplus-value of commodities, but in this case there is an all-important difference between constant capital and variable capital.

**Value-price**

The “value” of commodities in Marx’s theory is a complicated concept which has three interrelated dimensions— the *substance* of value (abstract labor), the *magnitude* of value

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3 As discussed in Chapter 1, the precise magnitude of C might change up until the time of the sale of the output, but C is still taken as given, because it still exists prior to the sale of the output and still comes from circulation.
(socially-necessary labor-time), and the necessary form of appearance of value (money and prices) (see the titles of the sections of Chapter 1 of Volume 1 of Capital). After Section 3 of Chapter 1, the “value” of commodities without further attribution usually refers to the third aspect of the form of appearance of value in terms of money and prices. For example, in the key Chapter 7 of Volume 1, in which Marx’s basic theory of surplus-value is presented, the “value” of the cotton and the yarn is always stated in terms of shillings (e.g. 15 shillings, 30 shillings, etc.). In order to be as clear as possible, I will call the price form of appearance of value in units of money the “value-price” of commodities. Others (e.g. Shaikh) have called these prices “direct prices”. (See Moseley 2004 for an extensive discussion of this issue.)

Furthermore, the value-price of commodities as products of capital is different from the value-price of simple commodities (as analyzed by Marx in Part 1 of Volume 1). The difference has to do with the “transferred value” component of the value-price of commodities. The “transferred value” component of the value-price of simple commodities is proportional to the labor-time required to produce the means of production, but the “transferred value” component of the value-price of commodities produced by capital is the actual constant capital advanced to purchase the means of production (i.e. the same constant capital which is taken as given as a component of the cost price in equation (2)), which is equal to the price of production of the means of production, and in general is not proportional to the labor-time required to produce the means of production. The means of production are purchased with constant capital, and thus the labor-time required to produce the means of production has already been represented as this quantity of money capital (even if somewhat misrepresented), and it is this quantity of already existing money capital advanced that becomes the first component of the value-price of commodities, even though this actual quantity of money capital is not equal to the value of the
means of production, but is instead equal to their prices of production. This previously existing money constant capital is transferred directly, as a given quantity of money capital, to the value-price of commodities produced by capital. Since the quantity of constant capital transferred cannot be greater than the original quantity of constant capital advanced, constant capital cannot be a source of surplus-value.\footnote{Other authors who have emphasized this difference between the value of commodities as “products of capital” and the value of simple commodities include Wolff-Robers-Calleri, Kliman-McGlone, and Ramos.}

In contrast, the given variable capital plays an entirely different role in the determination of value and surplus-value. The given variable capital is not transferred to the value-price of the product; i.e. variable capital does not become one component of the value-price. Instead, variable capital is replaced by the living labor that it purchases, and this living labor, when put to work, produces more value than it is paid. One part of this new value produced by labor allows capitalists to recover their variable capital paid to workers, and the other part becomes the surplus-value of capitalists.

Therefore, the total value-price of commodities produced by the total social capital in a year is the sum of two components: the given constant capital ($\overline{C}$), which existed previously (“old value”) and which is transferred to the value of the output, and the “new-value” (in money terms) produced by the labor of the current period ($N$):

\begin{equation}
P = \overline{C} + N
\end{equation}

In this way, the given actual constant capital becomes a determining factor of the total value-price of the commodities produced.

The new-value component of the value-price of commodities ($N$) (in units of money) is in turn determined by the product of the quantity of current socially necessary labor-time
(L) (in units of abstract labor-hours) and the (money) new-value produced *per hour of abstract labor* (m): \(^5\)

\[ N = mL \]

This is the key assumption in Marx’s labor theory of value: that the money new-value produced in the current period in the economy as a whole is proportional to the quantity of abstract labor in the economy as a whole. \(^6\)

With regard to the proportionality factor \( m \), Marx assumed throughout *Capital* that money is a commodity (e.g. gold). In this case, \( m \) is assumed to be determined by the quantity of gold produced per hour (e.g. 0.5 shillings per hour in Marx’s examples in the key Chapter 7 of Volume 1). The product of gold labor is directly and immediately money value, a quantity of money value equal to its own physical amount. An hour of abstract labor in all other industries is assumed to produce the *same quantity of money value* (m) as one hour of abstract labor in the gold industry. The difference between gold labour and all other labor is that one hour of abstract labor in the gold industry produces actual money value directly, as money itself, whereas one hour of abstract labor in all other industries produces the same amount of money value in the form of the prices of commodities, which still have to be converted into actual money value through sale. I argue in Chapter 5 (and in Moseley 2005) that the transformation of values into prices of production in Volume 3 does not affect the

\(^5\) The proportionality factor \( m \) has been called by Foley and others the “monetary expression of labor-time” or the “MELT”; see Chapter 8.

\(^6\) This assumption is also the key difference between Marx’s theory and Sraffa’s theory. There is no new-value component of the prices of commodities in Sraffa’s theory; i.e. no new-value produced by current labor. Labor is considered in Sraffa’s theory only as a *cost*, not as a producer of value, and in this respect (cost) is no different from the material inputs.
value of \( m \). (I also discuss the determination of \( m \) in the case of contemporary non-commodity money in Moseley 2010).

Substituting equation (4) into equation (3), we obtain:

\[
P = C + mL
\]

Thus, the first component of the value-price of commodities produced by capital is “old value” – value that existed prior to the production process in which it was consumed, and exists at the beginning of this production process in the form of the money constant capital advanced to purchase the means of production. The previously existing money constant capital is transferred directly, as a quantity of money capital, to the value-price of the commodities produced by capital during this production process. The second component, by contrast, is “new value” – value that did not exist prior to any given production process, and which is instead the result of the labor of this production process.

Substituting equations (2) and (3) into equation (1), we obtain:

\[
S = P - K
\]

\[
S = (C + N) - (C + \bar{V})
\]

We can see that constant capital is a component of both the value-price and the cost price of commodities. Marx referred to this very important point as the “dual significance of constant capital” (C.III. 119-120). As a result of its “dual significance”, constant capital cancels out in the determination of the surplus-value, and thus equation (6) simplifies to:

\[
S = N - \bar{V}
\]

\[
S = mL - \bar{V}
\]

Thus, according to this theory, the quantity of surplus-value is determined by the difference between the new-value produced by workers and the variable capital they are paid.
In this way, the given actual magnitude of variable capital becomes a determining factor (inversely) of the actual total surplus-value produced. By contrast, the quantity of surplus-value does not depend on the constant capital invested in means of production. The given constant capital is a determinant of the value-price of commodities, but it is not a determinant of surplus-value.

Similar to constant capital, the given actual variable capital is equal to the price of production of the means of subsistence purchased by workers with the variable capital, which in general is not proportional to the labor-time required to produce these means of subsistence. Nonetheless, the actual quantity of variable capital is still subtracted from the new-value produced in order to determine the actual surplus-value produced. Otherwise, variable capital and surplus-value of commodities would refer to hypothetical quantities (proportional to the labor-times required to produce wages goods and surplus goods, respectively), not actual quantities of money capital. To the contrary, Marx’s theory of surplus-value is about the actual quantity of surplus-value and actual quantities of money capital from the very beginning, not hypothetical quantities of surplus-value and variable capital that would later have to be transformed to actual quantities.
Average worker

Marx’s theory of surplus-value is presented in Chapter 7 of Volume 1 in terms of an average worker, which represents the working class as a whole, and whose average working day is divided into two parts – necessary labor ($NL$) and surplus-value ($SL$). Necessary labor is defined in Chapter 9 of Volume 1 as the number of hours of abstract labor that it takes the average worker to produce money new-value that is equal to the average variable capital that is paid to the worker per day ($V_i$); algebraically: $NL_i = V_i / m$. The remainder of the working day is surplus labor ($SL_i = L_i - NL_i$), i.e. the labor-time in which the money new-value produced by the worker no longer goes to reproduce an equivalent of the variable capital paid to the worker, but instead becomes the surplus-value of capitalists.

Interpreting equation (5) in terms of the average worker and substituting these definitions of $NL$ and $SL$, the surplus-value produced by the average worker per day is determined by:

\[
S_i = m L_i - \overline{V}_i = m L_i - m (NL_i) = m (L_i - NL_i) = m (SL_i)
\]

Therefore, the main conclusion of Marx’s theory is that the quantity of surplus-value produced by the average worker per day is proportional to the worker’s surplus labor time, determined in this way.

Marx’s theory of surplus-value is represented graphically in Figure 1. The horizontal axis is $L$, the quantity of labor-hours in the average working day. The variables on the vertical axis are: (1) variable capital, which is represented by the horizontal line, because wages are assumed to be paid by the day, so that the amount of wages per working day in constant, independent of the length of the working day; (2) new-value, which is represented by the upward sloping line, with slope = $m$; and (3) surplus-value, which is represented by the vertical distance between the new-value line and the variable capital line at any given
length of the working day. We can clearly see that, under these assumptions, the amount of surplus-value varies directly with the length of the working day, and inversely with the given variable capital.

**Figure 1 should go about here**

Marx assumed wages by the day in his theory of surplus-value because almost all wages were by the day in the 19th century (and up until the Great Depression; Henry Ford’s famous offer in 1913 was of course for $5 a day). But Marx’s theory of surplus-value applies equally as well to wages by the hour, the main form of wages today. In this case, each hour of labor would be divided into necessary labor (the number of minutes required for the average worker to produce new-value equal to the hourly wage) and surplus labor (the remainder of each hour which produces surplus-value). Necessary labor and surplus labor per day could then be obtained by adding up the hourly components. The fundamental conclusion is not affected – surplus-value is proportional to the workers’ surplus labor. Workers produce more value per hour than the hourly wage, just like workers produce more value per day than the daily wage (as in equation (8)).

The determination of surplus-value with wages by the hour is represented graphically in Figure 2, with the horizontal variable capital line replaced with an upward sloping line, with slope equal to the hourly wage rate, and this variable capital line is always underneath the new-value line because wages per hour are less than new-value produced per hour. Surplus-value is still the vertical distance between the new-value line and the (now upward sloping) variable capital line. The relation between the length of the working day and the amount of surplus-value is still positive, although not as strong as before, because a longer working day also means more wages (and may mean time-and-a-half for overtime hours, which would increase the slope of the variable capital line during the overtime hours).
This same theory of surplus-value applies to each and every worker in the capitalist economy as a whole. Therefore, one can aggregate this theory over all workers, or as a simplification, multiply the average surplus-value produced per worker per day times the total number of workers employed \((n)\). Then, in order to obtain the total annual surplus-value produced in the economy as a whole in a year, the surplus-value produced by all workers in a day is multiplied by the average number of working days per year \((d)\). Thus we have:

\[
S = (d \ n \ m \ SL_i)
\]

This then is Marx’s “surplus labor” theory of surplus-value. It explains the actual total annual surplus-value produced in the capitalist economy as a whole, and it concludes that the actual total surplus-value is proportional to the total amount of surplus labor of workers, with \(m\) as the factor of proportionality (i.e. each hour of surplus labor produces \(m\) amount of money surplus-value). This is the main conclusion of Volume 1. Most of the rest of Volume 1 is concerned with the main ways to increase surplus-value by increasing surplus labor: lengthening the working day and increasing the intensity of labor (absolute surplus-value) and reducing necessary labor through technological change (relative surplus-value).

2. Theory of the distribution of surplus-value and prices of production

I argued in Chapter 1 that the main subject of Volume 3 is the distribution of surplus-value, or the division of the predetermined total surplus-value into individual parts (see Chapter 3 for an extensive discussion of this issue).

Prices of production

The first and most fundamental aspect of the distribution of surplus-value, presented in Part 2 of Volume 3, is the equalization of the rate of profit across industries and the determination of prices of production. This was the “chief stumbling block” of Ricardo’s labor theory of value, and it was incumbent on Marx to explain how his labor theory of value (and surplus
labor theory of surplus-value) could solve this important problem and could be reconciled with the empirical tendency toward an equal rate of profit across industries. That is the objective of Marx’s theory of prices of production. According to Marx’s theory, the price of production in each industry \( (PP_i) \) is determined by the sum of two components: the cost price \( (K_i) \), which is the sum of the constant capital and variable capital consumed in the industry \( (K_i = C_i + V_i) \), and the average profit for the industry, which is determined by the product of the general rate of profit \( (R) \) and the total stock of capital invested in that industry \( (\bar{M}_i) \):

\[
PP_i = K_i + R \bar{M}_i
\]

(10)

We can see that Marx’s prices of production are not unit prices, in contrast to Sraffian theory, but are instead the sum of the total annual costs in an industry plus the average annual profit. A better name for Marx’s prices of production would be “gross annual industry revenue”. This is another fundamental difference between Marx’s theory and Sraffa’s theory – they are not even about the same micro price variables. One could derive unit prices from Marx’s prices of production, by dividing the price of production (gross annual industry revenue) in each industry by the quantity of annual output produced in that industry. But it is not clear why one would want to do that. The main point about prices of production is to explain how Marx’s labor theory of value is compatible with the tendency to equalize the rate of profit across industries. Unit prices are not necessary for this explanation, nor do they add anything to it.\(^7\)

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\(^7\) Marx briefly discussed unit prices in Section 1 of the “Results” manuscript (C.I: 957-71). In this manuscript, Marx assumes that prices = values, but the point he makes about the determination of unit prices remains true after prices of production have been determined – unit prices for each commodity are determined by dividing the price of production of that industry (i.e. the gross annual industry revenue) by the quantity of annual output produced.
Cost price

The cost price \((\bar{K}_i)\) and the total stock of capital invested \((\bar{M}_i)\) in each industry are taken as given, just as the total cost price \((\bar{K})\) and the total capital invested \((\bar{M})\) are taken as given in the Volume 1 theory of the total surplus-value, and for the same reasons: the individual amounts of money constant capital and variable capital invested in each industry are advanced in the sphere of circulation, prior to production, just as the total amounts of money constant capital and variable capital for the economy as a whole. Similarly, the inputs to individual industries are commodities with already existing prices, just as are the inputs to capitalist production as a whole. Therefore, these individual amounts of money constant capital and variable capital and prices of the commodity inputs already exist, prior to the production of the output, and are taken as given as such in the Volume 3 theory of prices of production, as an empirical fact. These already existing and presupposed quantities of money capital and prices of the inputs are determining factors of the prices of production of commodities in Volume 3, just as they are determining factors of the total value and surplus-value of commodities in Volume 1. In order to indicate that the cost price is taken as given for this reason, I will continue to use a bar over these variables, as in the prior theory of the total surplus-value.

And the crucial point with respect to the controversy over the “transformation problem” is that the industry quantities of \(K_i\) and \(M_i\) that are taken as given in Marx’s theory of prices of production are the same quantities of the total \(K\) the total \(M\) that are taken as given in the theory of the total surplus-value in Volume 1 - the actual quantities of money capital advanced in the real capitalist economy to purchase means of production and labor-power. The only difference is that, in Volume 3, the actual quantities of constant capital and variable capital advanced are disaggregated into individual industries. The sums of the individual quantities of constant capital and variable capital that are taken as given in the Volume 3 theory of prices of production are by definition equal to the total quantities of constant and variable capital that are taken as given in the Volume 1 theory of the total
surplus-value. We will see in Chapter 4 (Section x) that Marx stated and assumed many times that “the cost price is the same” in the determination of both value-prices and prices of production.

To clarify this crucial point further: applying equation (3) for the value-price of commodities to individual industries, we obtain the familiar equation:

\[(11) \quad P_i = C_i + N_i\]

As discussed above, the new-value component of the value-price of commodities is divided into two parts: one part which replaces the money variable capital paid for labor-power and the other part which is surplus-value:

\[(12) \quad N_i = \bar{V}_i + S_i\]

Substituting equation (12) into equation (11), we obtain:

\[(13) \quad P_i = \bar{C}_i + \bar{V}_i + S_i\]

which (combining \(\bar{C}_i + \bar{V}_i\)) simplifies to:

\[(14) \quad P_i = \bar{K}_i + S_i\]

Comparing equation (14) for value-prices and equation (10) above for prices of production, we can see that the cost price component is the same for both value-prices and prices of production – the actual quantities of money capital advanced and consumed in the real capitalist economy. The only component that is different is the second component – whether surplus-value or profit.

This is the reason why the quantities of constant capital and variable capital do not change, or do not have to be transformed, in the transition from the macroeconomic theory of the total surplus-value in Volume 1 to the microeconomic theory of individual prices of production in Volume 3 - because the same quantities of constant capital and variable capital are taken as given in both stages of the theory - the actual quantities of money capital advanced and consumed in the real capitalist economy. In other words, these actual given quantities of money constant capital and variable capital “remain invariant” in the transition from the macro theory of the total surplus-value in Volume 1 to the micro theory of the
individual parts of surplus-value in Volume 3. It is for this reason that Marx did not “fail to transform these inputs” - because the inputs do not have to be transformed, but instead remain invariant, as the actual given quantities of money capital advanced and consumed in the real capitalist economy.

We can see that the inputs of constant capital and variable capital are not determined simultaneously with the prices of production of the outputs. Instead, the inputs of constant capital and variable capital are taken as given in the sequential determination of the prices of production of the outputs.

**Rate of profit**

Returning to equation (8), the second component of prices of production, the average profit is equal to the product of the general rate of profit and the stock of capital invested in each industry. The general rate of profit is itself determined by the ratio of the actual total surplus-value, which is determined in Volume 1, to the actual total stock of capital invested, which is taken as given:

\[
R = \frac{S}{\overline{M}}
\]

In this way, the total surplus-value that is produced in the economy as a whole (determined in Volume 1) is distributed across individual industries according to the relative proportion of actual capital invested in each industry. Marx called this mode of distribution of the total surplus-value a kind of “capitalist communism”. (TSV.II: xxx)

Since the numerator in the rate of profit S is the actual total surplus-value, and the denominator \(\overline{M}\) is the actual total stock of capital invested, equal to the prices of production of the inputs purchased, the rate of profit determined in this way is the actual price rate of profit, not a hypothetical “value rate of profit” (in which both S and \(\overline{M}\) are proportional to the labor-values of particular bundles of goods), which later has to be “transformed” into the actual price rate of profit. The rate of profit in Marx’s theory is the price rate of profit from the beginning. Marx is accused of mistakenly using the “value rate of profit” to determine
prices of production; but this accusation is false. Marx did not use the “value rate of profit” to determine prices of production, because there is no “value rate of profit” in his theory for the economy as a whole. There are hypothetical “value rates of profit” for individual industries as a pedagogical device to explain the difference between the surplus-value produced in an individual industry and the profit appropriated in that industry; but since in the aggregate total profit = total surplus-value, there is no aggregate “value rate of profit” that differs from the aggregate price rate of profit. The rate of profit in Marx’s theory of prices of production is the actual price rate of profit.

We can also see that the rate of profit is not determined simultaneously with the prices of production of the output. Instead, the rate of profit is determined prior to prices of production, and taken as given in the sequential determination of prices of production. This method of determination of the rate of profit is the key link between Marx’s theory of the total surplus-value in Volume 1 and his theory of prices of production in Volume 3.

Finally, we can also see from the above summary that Marx’s theory of the production and distribution of surplus-value is about a “single system” – the actual capitalist economy – which is first analyzed at the aggregate level and then analyzed at the level of individual industries. The general rate of profit is determined at the aggregate level and then taken as given at the level of individual industries. The total amounts of constant capital and variable capital are taken as given at the aggregate level, and the individual amounts of constant capital and variable capital are taken as given at the industry level. At the aggregate level, the prices of individual commodities are not really considered. Marx made the provisional assumption in Volume 1 that the prices of individual commodities are equal to their value-prices, but it is not assumed commodities actually exchange at their value-prices. Individual commodities exchange only at prices of production, which are explained in Volume 3.
3. Marx’s two aggregate equalities always satisfied

An important implication of this “macro-monetary” interpretation is that Marx’s two aggregate equalities (total price of production = total value-price and total profit = total surplus-value) are *always both true simultaneously*, as Marx claimed. These two aggregate equalities are not true only for the special case of equal compositions of capital across industries, but are also true for the general case of unequal compositions of capital. These two aggregate equalities follow of necessity from Marx’s logical method of determination of the general rate of profit and prices of production, as discussed above.

Because the general rate of profit is determined as the ratio of the predetermined total surplus-value to the total capital advanced, the sum of all individual profits must of necessity be equal to the predetermined total surplus-value:

\[ \Sigma \Pi_i = \Sigma r M_i = R \Sigma M_i = R M = (S/M) M = S \]

Similarly, because the quantities of constant capital and variable capital that are taken as given in the determination of prices of production in Volume 3 are the same as the quantities of constant capital and variable capital that are taken as given in the determination of the total price, the sum of all individual prices of production must of necessity be equal to the total value-price as determined in Volume 1:

\[ \Sigma PP_i = \Sigma [ (C_i + V_i) + R M_i ] \]
\[ = \Sigma C_i + \Sigma V_i + R \Sigma M_i \]
\[ = C + V + S = C + N \]
\[ = P \]

In other words, one does not have to “choose an invariance condition”, i.e. choose only one of these two aggregate equalities to be true. All the key aggregate quantities in Marx’s theory - constant capital, variable capital, and surplus-value - remain invariant in the transition from the macroeconomic theory in Volume 1 to the microeconomic theory in Volume 3, and thus both aggregate equalities are always satisfied, as Marx argued.
Appendix: Comparison of interpretations

This short Appendix presents the following table, which compares the “macro-monetary-sequential” interpretation of Marx’s theory presented in this book with the Sraffian interpretation.

Table should be placed here (see two pages later)

We can see first of all the striking differences in the two interpretations of Volume 1, especially the different variables to be determined – individual unit labor-values vs. the total money surplus-value (i.e. total $\Delta M$) for the economy as a whole. In addition, the initial givens are fundamentally different – physical quantities (input-output and labor coefficients and the real wage) vs. quantities of money capital and socially-necessary labor time (and the MELT). Finally, the equations – a system of simultaneous equations vs. a sequence of logically deduced equations.

The differences with respect to Volume 3 are almost as striking. The initial givens are again fundamentally different – the same physical quantities plus individual labor-values vs. the total money surplus-value, the general rate of profit, and individual quantities of money capital. The variables determined appear to be more similar – both interpretations determine “prices of production” – but there is a significant difference between them, as discussed in the chapter. In my interpretation of Marx’s theory, prices of production are defined are “gross annual industry revenue” (the sum of total industry costs plus the average profit), whereas prices of production in the Sraffian interpretation
are defined as unit prices (i.e. price per unit of output). And the equations are again completely different – a system of simultaneous equations vs. independent equations for each industry, which are connected by the predetermined general rate of profit. One can also see from the second Sraffian interpretation equation for prices of production (the more common one in recent decades) that the Sraffian interpretation of Marx’s theory eliminates labor-values altogether, and turns it into Sraffian theory, which determines (unit) prices of production and the rate of profit directly from the given physical quantities.

Overall, all these differences can be summarized succinctly as the bottom line shows: the logical method of simultaneous determination vs. the logical method of sequential determination. Chapter 7 discusses these differences further.
Macro-monetary-
Sequential
Interpretation

Sraffian
Interpretation

VOLUME 1

Total economy

Individual value system

Givens

- total money capital \( M = C + V \)
- total current labor \( L \)
- money new-value per hour \( m \)

- input-output coefficients \( a_{ij} \)
- labor coefficients \( l_i \)
- real wage \( b_j \)

Variables determined

- total value-price \( P \)
- total surplus-value \( S \)
- general rate of profit \( R \)

- individual unit labor-values \( \lambda_i \)
- value rate of surplus-value \( s \)

Equations of
determination

\[
\begin{align*}
P &= C + N = C + mL \\
S &= N - V = mL - V = mL_s \\
R &= S / M
\end{align*}
\]

\[
\lambda_i = \lambda_j a_{ij} + l_i
\]

VOLUME 3

Individual industries

Individual price system

Givens

- total surplus-value \( S \)
- rate of profit \( R \)
- industry money capitals \( M_i = C_i + V_i \)

- input-output coefficients \( a_{ij} \)
- real wage \( b_j \)
- labor coefficients \( l_i \)
- unit labor-values \( \lambda_i \)

Variables determined

- industry prices of production \( PP_i \)

- unit price-labor coefficients \( z_i \)

- unit prices of production \( pp_i \)

- price rate of profit \( r \)

Equations of
Determination

\[
(PP_i) = (C_i + V_i) + R (M_i)
\]

\[
pp_i = (z_j \lambda_j a_{ij} + z_j \lambda_j b_l l_i) (1+r)
\]

\[
pp_i = (p_j a_{ij} + p_j b_j l_i) (1+r)
\]

Method of
determination

sequential

simultaneous